MARVELLOUS EVENT ON GEOMETRIC ANALYSIS

BĘDLEWO, OCTOBER 8 – 13, 2023

Sunday 7 pm bus transfer from Poznań train station to Będlewo, dinner 7 – 9 pm

	Monday	Tuesday	Wednesday	Thursday	Friday
08.15 – 09.15	breakfast				
09.15 – 09.30	opening				
09.30 – 10.30	**Hajłasz**	**Duzaar**	**Kuwert**	**Hencl**	**Rivière**
10.30 – 11.00	coffee				
11.00 – 12.00	**Kałamajska**	**Gastel**	**Bögelein**	**Schikorra**	**Hélein**
12.30 – 14.00	lunch				
14.00 – 15.00	**von der Mosel**	**Struwe**		**Koskela**	
15.00 – 15.30	coffee			coffee	
15.30 – 16.30	**Blatt**	lightning talks poster session	excursion	**Zhong**	
16.30 – 16.45	break			break	
16.45 – 17.45	**Peltonen**			**Kusner**	
	18.00 barbecue [if weather permits]	18.00 dinner	19.00 conference dinner	18.00 dinner	

Simon Blatt (*University of Salzburg*)

The L^2-gradient flow of O'Hara energies

In this talk, we present new findings concerning the L^2 gradient flow associated to the O'Hara energies. Depending on parameter choices, this family of energies gives rise to various evolution equations, all with a non-local character. For a specific one-parameter subfamily including the Möbius energy, the underlying evolution equation is critical.

While we have a satisfactory understanding of the behavior of solutions in the non-degenerate case, in the general case even fundamental questions, such as short-term existence, have remained open until now. In this presentation, we will demonstrate how to construct a weak solution for the underlying evolution equation, establish long-time existence for the flow, and investigate its asymptotic properties.

This is joint work with Nicole Vorderobermeier.

Verena Bögelein (*University of Salzburg*)

Gradient regularity for widely degenerate elliptic and parabolic systems

In this talk we consider widely degenerate elliptic systems of the form

$$\operatorname{div}\left((|Du|-1)_+^{p-1}\frac{Du}{|Du|}\right)=f, \qquad p>1$$

and its parabolic couter-part. Such PDEs are motivated for instance by congested traffic dynamics problems or models of gas filtration with nonlinear effects. Continuity properties of the gradient have been investigated in the scalar elliptic setting by Santambrogio & Vespri and Colombo & Figalli. In this talk we establish gradient regularity in the vector valued case for the elliptic as well as for the parabolic problem. More precisely, we can show that $\mathcal{K}(Du)$ is continuous for any continuous function $\mathcal{K}\colon \mathbb{R}^{Nn} \to \mathbb{R}^{Nn}$ vanishing on the set $\{\xi \in \mathbb{R}^{Nn} : |\xi| \leq 1\}$. This is joint work with F. Duzaar, R. Giova and A. Passarelli di Napoli.

Frank Duzaar (*University of Salzburg*)

Gradient estimates for a class of doubly nonlinear parabolic equations

In this talk, we address the local behavior of nonnegative weak solutions of the doubly nonlinear parabolic equation

$$\partial_t u^q - \operatorname{div}\left(|Du|^{p-2}Du\right) = 0.$$

In particular, we will discuss Hölder estimates for the gradient of weak solutions in the supercritical fast diffusion regime $0 < p-1 < q < \frac{N(p-1)}{(N-p)_+}$, where N is the space dimension. Two main components of these regularity estimates are a time-insensitive Harnack inequality specific to this regime and Schauder estimates for the parabolic p-Laplace equation.

These results are the outcome of a collaboration with Verena Bögelein (Salzburg), Ugo Gianazza (Pavia), Naian Liao (Salzburg) and Christoph Scheven (Duisburg-Essen).

Andreas Gastel (*University of Duisburg-Essen*)

Harmonic maps and micropolar elastic solids and shells

There are Cosserat type models for elastic solids or shells with additional "microrotational" degrees of freedom that couple a (p-)harmonic map equation with some elasticity equation. We report on our (and other people's) attempts to understand the regularity for the coupled system with the help of harmonic map theory. This is based on joint works with Vanessa Hüsken and Patrizio Neff.

Piotr Hajłasz (*University of Pittsburgh*)

Approximation of mappings with derivatives of low rank

My talk is based on two recent joint papers with Paweł Goldstein.

Jacek Gałęski in 2017, in the context of his research in geometric measure theory, formulated the following conjecture:

Conjecture. Let $1 \leq m < n$ be integers and let $\Omega \subset \mathbb{R}^n$ be open. If $f \in C^1(\Omega, \mathbb{R}^n)$ satisfies $\operatorname{rank} Df \leq m$ everywhere in Ω, then f can be uniformly approximated by smooth mappings $g \in C^\infty(\Omega, \mathbb{R}^n)$ such that $\operatorname{rank} Dg \leq m$ everywhere in Ω.

One can also modify the conjecture and ask about a local approximation: smooth approximation in a neighborhood of any point. These are very natural problems with possible applications to PDEs and Calculus of Variations. However, the problems are difficult, because standard approximation techniques like the one based on convolution do not preserve the rank of the derivative. It is a highly nonlinear constraint, difficult to deal with.

In 2018 Goldstein and Hajłasz obtained infinitely many counterexamples to this conjecture. Here is one:

Example. There is $f \in C^1(\mathbb{R}^5, \mathbb{R}^5)$ with $\operatorname{rank} Df \leq 3$ that cannot be locally and uniformly approximated by mappings $g \in C^2(\mathbb{R}^5, \mathbb{R}^5)$ satisfying $\operatorname{rank} Dg \leq 3$.

This example is a special case of a much more general result and the construction heavily depends on algebraic topology including the homotopy groups of spheres and the Freudenthal suspension theorem.

More recently Goldstein and Hajłasz proved the conjecture in the positive in the case when $m = 1$. The proof is based this time on methods of analysis on metric spaces and in particular on factorization of a mapping through metric trees.

The method of factorization through metric trees used in the proof of the conjecture when $m = 1$ is very different and completely unrelated to the methods of algebraic topology used in the construction of counterexamples. However, quite surprisingly, both techniques have originally been used by Wenger and Young as tools for study of Lipschitz homotopy groups of the Heisenberg group, a problem that seems completely unrelated to problems discussed in this talk.

Frédéric HÉLEIN (*Paris Cité University*)

Kaluza-Klein theories without fiber bundle structure hypothesis

I will present a theory based on a variational principle, the critical points of which allows us to build solutions of Einstein-Maxwell or, more generally, Einstein-Yang-Mills systems, in the spirit of Kaluza-Klein theories. The novelty is that the fibration hypothesis is not necessary: fields are given on a 'space-time' Y of dimension $4 + r$ without any structure a priori, where r is the dimension of the structure group. If this group is compact and simply connected, a solution allows to construct a manifold X of dimension 4 which can be interpreted as the physical space-time, in such a way that Y acquires a principal bundle structure over X and produces solutions of an Einstein-Yang-Mills system. If the structure group is $U(1)$, a case which corresponds to the Einstein-Maxwell system, the situation is slightly degenerate are supplementary hypotheses are required.

Stanislav HENCL (*Charles University in Prague*)

Injectivity in second-gradient Nonlinear Elasticity

We study injectivity for models of Nonlinear Elasticity that involve the second gradient. We assume that $\Omega \subset \mathbb{R}^n$ is a domain, $f \in W^{2,q}(\Omega, \mathbb{R}^n)$ satisfies $|J_f|^{-a} \in L^1$ and that f equals a given homeomorphism on $\partial\Omega$. Under suitable conditions on q and a we show that f must be a homeomorphism. As a main new tool we find an optimal condition for a and q that imply that $\mathcal{H}^{n-1}(\{J_f = 0\}) = 0$ and hence J_f cannot change sign.

This is a joint result with D. Campbell, A. Menovschikov and S. Schwarzacher.

Agnieszka Kałamajska (*University of Warsaw*)

Density results and trace operator for weighted Dirichlet and Sobolev spaces defined on the half-line and applications to the interpolation theory

We give an analytic description for the completion of $C_0^\infty(\mathbf{R}_+)$ in Dirichlet space $D^{1,p}(\mathbf{R}_+,\omega) := \{u : \mathbf{R}_+ \to \mathbf{R} : u$ is locally absolutely continuous on $\mathbf{R}_+$ and $\|u'\|_{L^p(\mathbf{R}_+,\omega)} < \infty\}$, for given continuous positive weight ω defined on $\mathbf{R}_+$, where $1 < p < \infty$. The conditions are described in terms of the modified variants of the B_p conditions due to Kufner and Opic from 1984, which in our approach are focusing on the integrability of $\omega^{-p/(p-1)}$ near zero or near infinity. We also propose applications of our results to: obtaining new variants of Hardy inequality, interpretation of boundary value problems in ODEs defined on the half-line with solutions in $D^{1,p}(\mathbf{R}_+,\omega)$, new results from complex interpolation theory dealing with interpolation spaces between weighted Dirichlet spaces, and for deriving new Morrey type embedding theorems for our Dirichlet space. Similar results were obtained for weigted Sobolev space $W^{1,p}(\mathbf{R}_+,t^\beta) = \{u \in L^p(\mathbf{R}_+,t^\beta) : u' \in L^p(\mathbf{R}_+,t^\beta)\}$, where $\beta \in \mathbf{R}$.

The talk will be based on my joint works obtained together with Claudia Capone and Radosław Kaczmarek.

[1] R. Kaczmarek, A. Kałamajska, *Density results and trace operator in weighted Sobolev spaces defined on the half-line, equipped with power weights*, Journal of Approximation Theory 291 (2023). Art. No. 105896, https://doi.org/10.1016/j.jat.2023.105896

[2] C. Capone, A. Kałamajska, *Asymptotics, Trace, and Density Results for Weighted Dirichlet Spaces Defined on the Half-line,* Potential Analysis, published online 24 July 2023, https://doi.org/10.1007/s11118-023-10089-2

Pekka Koskela (*University of Jyväskylä*)

tba

Rob Kusner (*University of Massachusetts at Amherst*)

On Eigenspaces and Minimal Surfaces

Sharp dimension bounds for (Steklov and Laplacian) eigenspaces on Riemannian surfaces are used to study existence and classification problems for embedded minimal surfaces in the round ball B^n (with free boundary) and sphere S^n (compact without boundary). In particular, we construct orientable minimal surfaces embedded in B^3 of each topological type, with analogous results for nonorientable surfaces and $n = 4$.

Joint projects with Peter McGrath, as well as with Misha Karpukhin and Daniel Stern.

Ernst Kuwert (*University of Freiburg i. Br.*)

Curvature varifolds with orthogonal boundary

We consider the class of surfaces in a given container with orthogonal constraint on the boundary. We discuss the problem to estimate the area in terms of curvature energy in that class. As an application, we prove the existence of a surface which minimizes the curvature energy (joint work with Marius Müller).

Kirsi Peltonen (*Aalto University*)
Shapes in Action
We will discuss about our decade long journey to develop communication between mathematics, arts, design and architecture at Aalto University, Finland. As a concrete opening in education, we have launched a Math&Arts Minor that is open to all university students. We will describe some related interdisciplinary activities and research directions from the perspective of mathematics.

Tristan Rivière (*ETH Zurich*)
tba

Armin Schikorra (*University of Pittsburgh*)
Regularity results for n-Laplace systems with antisymmetric potential
n-Laplace systems with antisymmetric potential are known to govern geometric equations such as n-harmonic maps between manifolds and generalized prescribed H-surface equations. Due to the nonlinearity of the leading order n-Laplace and the criticality of the equation they are very difficult to treat. I will discuss some progress we obtained, combining stability methods by Iwaniec and nonlinear potential theory for vectorial equations by Kuusi-Mingione.

Joint work with Dorian Martino.

Michael Struwe (*ETH Zurich*)
Plateau flow, or the heat flow for half-harmonic maps
Using the interpretation of the half-Laplacian on S^1 as the Dirichlet-to-Neumann operator for the Laplace equation on the ball B, we devise a classical approach to the heat flow for half-harmonic maps from S^1 to a closed target manifold N in $\mathbb{R}^n$, recently studied by Wettstein, and for arbitrary finite-energy data we obtain a result fully analogous to my result for the harmonic map heat flow of surfaces from 1985, and in similar generality.

When N is a smoothly embedded, oriented closed curve, the half-harmonic map heat flow may be viewed as an alternative gradient flow for a variant of the Plateau problem of disc-type minimal surfaces. In particular, the results invite to study the parametric Plateau problem without a monotonicity requirement.

Heiko von der Mosel (*RWTH Aachen University*)
Elastic knots
To model knotted springy wires we minimize in prescribed knot classes a total energy consisting of the classic Euler-Bernoulli bending energy and an additive repulsive potential. The ultimate goal is to characterize the shape of such minimizing knots for various knot classes. For that we send a prefactor of the repulsive potential to zero and analyze the limiting configurations – so-called *elastic knots*. For all torus knot classes $\mathcal{T}(2,b)$ we established the doubly-covered circle as the unique elastic knot, which confirms mechanical and numerical experiments. There are, however, instances when the numerical gradient flow seems to get stuck in different configurations exhibiting some symmetry.

To provide analytic support for these rare observations we use the symmetric criticality principle to find symmetric elastic knots exhibiting these symmetries.

In this talk we give a survey on the analytic results, show some of the numerical simulations obtained by Bartels, Riege, and Reiter, and by Schumacher, and address many open questions.

Xiao Zhong (*University of Helsinki*)

Variational problems with gradient constraints

I will talk about three different types of variational problems with gradient constraints. They arise from elastic-plastic torsion, hypersurfaces in the Lorentz-Minkowski spaces with given mean curvature and dimer models.